$$Z(n) = \sum_{r=N} x(r)\, y(n-r) \quad \checkmark$$

(5/10)

Proof:

$$Z_k = \frac{1}{N}\sum_{n=N} z(n) e^{-jk\omega_0 n}$$

$$= \frac{1}{N}\sum_{n=N}\left(\sum_{r=N} x(r)\, y(n-r)\right) e^{-jk\omega_0 n}$$

$$Z_k = \sum_{r=N} x(r) \underbrace{\left(\frac{1}{N}\sum_{n=N} y(n-r)\, e^{-jk\omega_0 n}\right)}_{Y_k e^{-jk\omega_0 r}} \quad \checkmark$$

Using time shifting property $y(n) \leftrightarrow Y_k$

$$y(n-r) \rightleftharpoons e^{-j\omega_0 n r} Y_k$$

$$Z_k = \frac{N}{N} \cdot \frac{1}{N}\sum_{r=N} x(r) e^{-jr\omega_0 n}\, Y_k$$

$$\boxed{Z_k = N\, X_k \times Y_k}$$

Proved.

$r=N$, ~~$Z_k = N \times X_k \times Y_k$~~

statement only

(7) Multiplication: $x(n) \leftrightarrow X_k$, $y(n) \leftrightarrow Y_k$

$$\boxed{z(n) = x(n)\, y(n) \longleftrightarrow Z_k = \sum_{r=N} X_r Y_{k-r}}$$

$Z_k \rightarrow$ is the periodic convolution between the two periodic sequences of Fourier Trans X_k and Y_k.

$\Rightarrow$

Consider the signal $z(n)$

$$z(n) = x_n y(n) = \sum_{r=N} X_r e^{jr\omega_0 n} \sum_{m=N} Y_m e^{jm\omega_0 n}$$

$$= \sum_{r=\langle N\rangle} X_r \sum_{m=\langle N\rangle} Y_m e^{j(m+r)\omega_0 n}.$$

now $m+r=K$.

$m = K-r$, $K \to r$ as $m \to 0$

and $K \to (r+N-1)$ as $m \to (N-1)$.

$$z(n) = \sum_{r=N} X_r \sum_{K=r}^{r+N-1} Y_{K-r} e^{jK\omega_0 n}.$$

$$z(n) = \sum_{K\langle N\rangle} \left(\sum_{r=N} X_r Y_{K-r} \right) e^{jK\omega_0 n} = \sum_{K\langle N\rangle} Z_K e^{jK\omega_0 n}$$

$$\boxed{Z_K = \sum_{r=N} X_r Y_{K-r}.}$$

8 First Difference

It uses first difference eqn $y(n) = x(n) - x(n-1)$

If $x(n)$ is periodic with period N,

since shifting $x(n)$ or linearly combining $x(n)$ with another periodic signal whose period is N always result in a periodic signal with period N.

So $x(n) \leftrightarrow X_K$

$$y(n) = x(n) - x(n-1)$$

$$\Rightarrow Y_K \leftrightarrow \left(1 - e^{-jK\omega_0}\right) X_K$$

Proof $x(n) \leftrightarrow X_k$

using time shifting $x(n-1) \leftrightarrow X_k e^{-jk\omega_0}$

using linearity

$x(n) - x(n-1) \leftrightarrow X_k - X_k e^{-kj\omega_0}$

(9) Running sum or Accumulation

$x(n) \leftrightarrow X_k$

$y(n) \leftrightarrow Y_k$

then $y_n = \sum_{k=-\infty}^{n} x(k) \longleftrightarrow Y_k = \left(\frac{1}{1-e^{-jk\omega_0}}\right) X_k,$

$k \neq 0$

The discrete-time fourier series coefficient Y_k of the running sum $y(n) = \sum_{k=-\infty}^{n} x(k)$ is finite valued and periodic only if $X_0 = 0$

Proof: Consider the running sum.

$$y(n) = \sum_{k=-\infty}^{n} x(k)$$

$$y(n) = x(n) + \sum_{k=-\infty}^{n-1} x(k)$$

$$y(n) = x(n) + y(n-1)$$

or $y(n) - y(n-1) = x(n)$.

using time shifting

$$Y_k - Y_k e^{-jk\omega_0} = X_k$$

$$\therefore Y_k = \frac{X_k}{1-e^{-jk\omega_0}}$$